a certain sum daily; this sum being fixed for each, but different for the two. After six weeks, the box was empty. Find the sum which each man drew daily from the box: knowing that A *alone* would have emptied it five weeks earlier than B *alone*.

12. Proposed by F. M. SHIELDS, Coopwood, Mississippi.

Three lads, A, B, and C, each climbed to the top of an upright pole; A's pole was 20 feet high, B's 60 feet, and C's pole was 100 feet high. They all started at the same time, and each climbed up a part of the way, at the same rate of speed per minute, and after each rested 5 minutes, they ascended to the tops of their respective poles, at the same rate of speed per minute, when they found that each had consumed the same length of time, 25 minutes each, (including the 5 minutes each rested on the way). How far up did each climb before resting? At what rates of speed per minute did they ascend?

13. Proposed by H C. WHITAKER, B. S., M. E., Professor of Mathematics, Manual Training School, Philadelphia.

Six city boys, Jim, Josh, Jerry, Jack, Jake and Jeorje went into the country to steal apples from a tree. While three kept watch, the other three climbed up and got what they wanted. Then they came down while the other three rascals went up and stole. The one that got most was one of the last to go up.

Each trio of theives took the same number and had each boy taken as many as he did take in *each of that number of pockets*, each trio would also have taken the same number and the tree would have lost 538 apples. As it was, Josh got more than Jack, but Jeorje got as many as Josh and Jack together, while Jake got twice as many as Jerry and two more than Jim. What were the names of the three that first kept watch? [Figures altered from problem in *Henkle's Notes and Queries*.]

[Solutions to these problems should be received by April 1st.]

GEOMETRY.

Conducted by B. F. FINKEL, Kidder, Missouri. All contributions to this department should be sent to him.

2. Show that $\frac{1}{2}\pi=\left[\frac{2.\ 4.\ 6.\ 8.\ 10}{1.\ 3.\ 5.\ 7.\ 9}\cdots\cdots\right]^2$, *Wallis's* expression for π.

[Selected from *Bowser's Trigonometry.*]

Solution by Professor ZERR. A. M., Principal of School, Staunton, Virginia.

All trigonometries solve the following:

$$\sin\theta=\theta\left[1-\frac{\theta^2}{\pi^2}\right]\left[1-\frac{\theta^2}{2^2\pi^2}\right]\left[1-\frac{\theta^2}{3^2\pi^2}\right]\left[1-\frac{\theta^2}{4^2\pi^2}\right]\cdots\cdots$$

Now let $\theta=\frac{\pi}{2}$. Then $1=\frac{\pi}{2}(1-\frac{1}{4})(1-\frac{1}{16})(1-\frac{1}{36})(1-\frac{1}{64})(1-\frac{1}{100})(1-\frac{1}{144})\cdots$

$$=\frac{\pi}{2}\cdot\frac{3}{4}\cdot\frac{15}{16}\cdot\frac{35}{36}\cdot\frac{63}{64}\cdot\frac{99}{100}\cdot\frac{143}{144}\cdots\cdot$$

$$=\frac{\pi}{2}\cdot\frac{3}{2.2}\cdot\frac{3.5}{4.4}\cdot\frac{5.7}{6.6}\cdot\frac{7.9}{8.8}\cdot\frac{9.11}{10.10}\cdot\frac{11.13}{12.12}\cdots\cdot$$

$$=\frac{\pi}{2}\cdot\frac{3^2.5^2.7^2.9^2.11^2}{2^2.4^2.6^2.8^2.10^2}\cdots\cdot$$

$$\therefore\ \frac{\pi}{2}=\left\{\frac{2.4.6.8.10}{1.3.5.7.9.11}\cdots\cdot\right\}^2$$

3. If A be the area of the circle inscribed in a triangle, A_1, A_2, A_3 the areas of the escribed circle, show that $\dfrac{1}{\sqrt{A}}=\dfrac{1}{\sqrt{A_1}}+\dfrac{1}{\sqrt{A_2}}+\dfrac{1}{\sqrt{A_3}}$.

[Selected from *Todhunter's Plane Trigonometry.*]

Solution by ROBERT J. ALEY, A. M., Professor of Mathematics in the Indiana State University, Bloomington, Indiana, and J. A. CALDERHEAD, Superintendent of School, Limaville, Ohio.

$r=\dfrac{S}{s}$, where r=radius of inscribed circle, S=area of triangle, and $s=\frac{1}{2}(a+b+c)$=half the sum of the sides of triangle. (*Todhunter's Plane Trigonometry*, Art. 248.)

Also $r_1=\dfrac{S}{s-a}$, $r_2=\dfrac{S}{s-b}$, $r_3=\dfrac{S}{s-c}$ where r_1, r_2, r_3 respectively, represent the radii of the escribed circles. (Art. 250).

$$\therefore A=\frac{\pi S^2}{s^2}, A_1=\frac{\pi S^2}{(s-a)^2}, A_2=\frac{\pi S^2}{(s-b)^2}, A_3=\frac{\pi S^2}{(s-c)^2}.$$

$$\therefore \frac{1}{\sqrt{A}}=\frac{s}{S\sqrt{\pi}}, \frac{1}{\sqrt{A_1}}=\frac{s-a}{S\sqrt{\pi}}, \frac{1}{\sqrt{A_2}}=\frac{s-b}{S\sqrt{\pi}}, \frac{1}{\sqrt{A_3}}=\frac{s-c}{S\sqrt{\pi}}.$$

$$\therefore \frac{1}{\sqrt{A_1}}+\frac{1}{\sqrt{A_2}}+\frac{1}{\sqrt{A_3}}=\frac{s-a}{S\sqrt{\pi}}+\frac{s-b}{S\sqrt{\pi}}+\frac{s-c}{S\sqrt{\pi}}=\frac{3s-(a+b+c)}{S\sqrt{\pi}}=\frac{s}{S\sqrt{\pi}}.$$

But $\dfrac{1}{\sqrt{A}}=\dfrac{s}{S\sqrt{\pi}}$. $\therefore \dfrac{1}{\sqrt{A}}=\dfrac{1}{\sqrt{A_1}}+\dfrac{1}{\sqrt{A_2}}+\dfrac{1}{\sqrt{A_2}}$. Q. E. D.

Also solved by *G. B. M. Zerr, P. S. Berg, P. H. Philbrick, J. R. Baldwin,* and *H. C. Whitaker.*

PROBLEMS.

17. **Proposed by ROBERT J. ALEY, A. M., Professor of Mathematics in the Indiana University, Bloomington, Indiana.**

Draw a circle bisecting the circumferences of three given circles.

18. **Proposed by Professor HENRY HEATON, Atlantic, Iowa.**

Through two given points to draw two circles tangent to a given circle.

19 **Proposed by J. A. CALDERHEAD, Superintendent of Schools, Limaville, Ohio.**

If any point be taken in the circumference of a circle, and lines be drawn from it to the three angles of an inscribed equilateral triangle, prove that the middle line so drawn is equal to the sum of the other two.

20. **Proposed by GEORGE BRUCE HALSTED, A. M., Ph. D., Member of the London Mathematical Society, and Professor of Mathematics in the University of Texas, Austin, Texas.**

Demonstrate by pure spherical geometry that spherical tangents from any point in the produced spherical chord common to two intersecting circles on a sphere are equal.